# CHAPTER 1

# DEFINITIONS AND UNITS

## 1.1 INTRODUCTION

The story of magnetism begins with a mineral called magnetite ($Fe_3O_4$), the first magnetic material known to man. Its early history is obscure, but its power of attracting iron was certainly known 2500 years ago. Magnetite is widely distributed. In the ancient world the most plentiful deposits occurred in the district of Magnesia, in what is now modern Turkey, and our word magnet is derived from a similar Greek word, said to come from the name of this district. It was also known to the Greeks that a piece of iron would itself become magnetic if it were touched, or, better, rubbed with magnetite.

Later on, but at an unknown date, it was found that a properly shaped piece of magnetite, if supported so as to float on water, would turn until it pointed approximately north and south. So would a pivoted iron needle, if previously rubbed with magnetite. Thus was the mariner's compass born. This north-pointing property of magnetite accounts for the old English word lodestone for this substance; it means "waystone," because it points the way.

The first truly scientific study of magnetism was made by the Englishman William Gilbert (1540–1603), who published his classic book *On the Magnet* in 1600. He experimented with lodestones and iron magnets, formed a clear picture of the Earth's magnetic field, and cleared away many superstitions that had clouded the subject. For more than a century and a half after Gilbert, no discoveries of any fundamental importance were made, although there were many practical improvements in the manufacture of magnets. Thus, in the eighteenth century, compound steel magnets were made, composed of many magnetized steel strips fastened together, which could lift 28 times their own weight of iron. This is all the more remarkable when we realize that there was only one way of making magnets at that time: the iron or steel had to be rubbed with a lodestone, or with

*Introduction to Magnetic Materials, Second Edition.* By B. D. Cullity and C. D. Graham

another magnet which in turn had been rubbed with a lodestone. There was no other way until the first electromagnet was made in 1825, following the great discovery made in 1820 by Hans Christian Oersted (1775–1851) that an electric current produces a magnetic field. Research on magnetic materials can be said to date from the invention of the electromagnet, which made available much more powerful fields than those produced by lodestones, or magnets made from them.

In this book we shall consider basic magnetic quantities and the units in which they are expressed, ways of making magnetic measurements, theories of magnetism, magnetic behavior of materials, and, finally, the properties of commercially important magnetic materials. The study of this subject is complicated by the existence of two different systems of units: the *SI* (*International System*) or *mks*, and the *cgs* (electromagnetic or *emu*) systems. The SI system, currently taught in all physics courses, is standard for scientific work throughout the world. It has not, however, been enthusiastically accepted by workers in magnetism. Although both systems describe the same physical reality, they start from somewhat different ways of visualizing that reality. As a consequence, converting from one system to the other sometimes involves more than multiplication by a simple numerical factor. In addition, the designers of the SI system left open the possibility of expressing some magnetic quantities in more than one way, which has not helped in speeding its adoption.

The SI system has a clear advantage when electrical and magnetic behavior must be considered together, as when dealing with electric currents generated inside a material by magnetic effects (eddy currents). Combining electromagnetic and electrostatic *cgs* units gets very messy, whereas using SI it is straightforward.

At present (early twenty-first century), the SI system is widely used in Europe, especially for soft magnetic materials (i.e., materials other than permanent magnets). In the USA and Japan, the cgs–emu system is still used by the majority of research workers, although the use of SI is slowly increasing. Both systems are found in reference works, research papers, materials and instrument specifications, so this book will use both sets of units. In Chapter 1, the basic equations of each system will be developed sequentially; in subsequent chapters the two systems will be used in parallel. However, not every equation or numerical value will be duplicated; the aim is to provide conversions in cases where they are not obvious or where they are needed for clarity.

Many of the equations in this introductory chapter and the next are stated without proof because their derivations can be found in most physics textbooks.

## 1.2 THE cgs–emu SYSTEM OF UNITS

### 1.2.1 Magnetic Poles

Almost everyone as a child has played with magnets and felt the mysterious forces of attraction and repulsion between them. These forces appear to originate in regions called poles, located near the ends of the magnet. The end of a pivoted bar magnet which points approximately toward the north geographic pole of the Earth is called the north-seeking pole, or, more briefly, the north pole. Since unlike poles attract, and like poles repel, this convention means that there is a region of south polarity near the north geographic pole. The law governing the forces between poles was discovered independently in England in 1750 by John Michell (1724–1793) and in France in 1785 by Charles Coulomb (1736–1806). This law states that the force $F$ between two poles is proportional

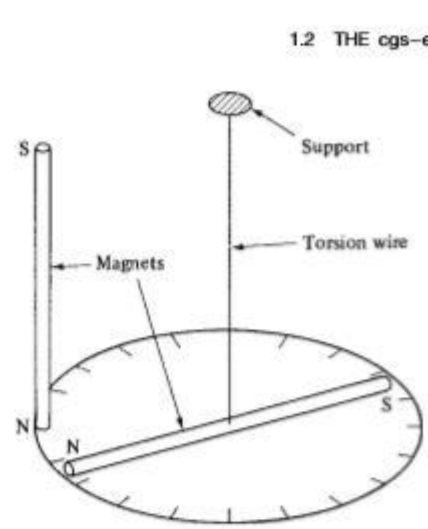


**Fig. 1.1** Torsion balance for measuring the forces between poles.

to the product of their pole strengths $p_1$ and $p_2$ and inversely proportional to the square of the distance $d$ between them:

$$F = k\frac{p_1 p_2}{d^2}. \tag{1.1}$$

If the proportionality constant $k$ is put equal to 1, and we measure $F$ in dynes and $d$ in centimeters, then this equation becomes the definition of pole strength in the cgs–emu system. A unit pole, or pole of unit strength, is one which exerts a force of 1 dyne on another unit pole located at a distance of 1 cm. The dyne is in turn defined as that force which gives a mass of 1 g an acceleration of 1 cm/sec$^2$. The weight of a 1 g mass is 981 dynes. No name has been assigned to the unit of pole strength.

Poles always occur in pairs in magnetized bodies, and it is impossible to separate them.[1] If a bar magnet is cut in two transversely, new poles appear on the cut surfaces and two magnets result. The experiments on which Equation 1.1 is based were performed with magnetized needles that were so long that the poles at each end could be considered approximately as isolated poles, and the torsion balance sketched in Fig. 1.1. If the stiffness of the torsion-wire suspension is known, the force of repulsion between the two north poles can be calculated from the angle of deviation of the horizontal needle. The arrangement shown minimizes the effects of the two south poles.

A magnetic pole creates a magnetic field around it, and it is this field which produces a force on a second pole nearby. Experiment shows that this force is directly proportional to the product of the pole strength and field strength or field intensity $H$:

$$F = kpH. \tag{1.2}$$

If the proportionality constant $k$ is again put equal to 1, this equation then defines $H$: a field of unit strength is one which exerts a force of 1 dyne on a unit pole. If an unmagnetized

[1]The existence of isolated magnetic poles, or *monopoles*, is not forbidden by any known law of nature, and serious efforts to find monopoles have been made [P. A. M. Dirac, *Proc. R. Soc. Lond.*, **A133** (1931) p. 60; H. Jeon and M. J. Longo, *Phys. Rev. Lett.*, **75** (1995) pp. 1443–1446]. The search has not so far been successful.

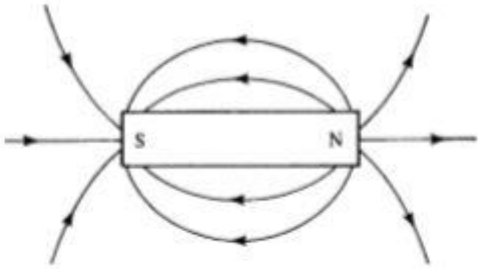


**Fig. 1.2** External field of a bar magnet.

piece of iron is brought near a magnet, it will become magnetized, again through the agency of the field created by the magnet. For this reason $H$ is also sometimes called the *magnetizing force*. A field of unit strength has an intensity of one *oersted* (Oe). How large is an oersted? The magnetic field of the Earth in most places amounts to less than 0.5 Oe, that of a bar magnet (Fig. 1.2) near one end is about 5000 Oe, that of a powerful electromagnet is about 20,000 Oe, and that of a superconducting magnet can be 100,000 Oe or more. Strong fields may be measured in kilo-oersteds (kOe). Another cgs unit of field strength, used in describing the Earth's field, is the *gamma* ($1\gamma = 10^{-5}$ Oe).

A unit pole in a field of one oersted is acted on by a force of one dyne. But a unit pole is also subjected to a force of 1 dyne when it is 1 cm away from another unit pole. Therefore, the field created by a unit pole must have an intensity of one oersted at a distance of 1 cm from the pole. It also follows from Equations 1.1 and 1.2 that this field decreases as the inverse square of the distance $d$ from the pole:

$$H = \frac{p}{d^2}. \tag{1.3}$$

Michael Faraday (1791–1867) had the very fruitful idea of representing a magnetic field by "lines of force." These are directed lines along which a single north pole would move, or to which a small compass needle would be tangent. Evidently, lines of force radiate outward from a single north pole. Outside a bar magnet, the lines of force leave the north pole and return at the south pole. (Inside the magnet, the situation is more complicated and will be discussed in Section 2.9) The resulting field (Fig. 1.3) can be made visible in two dimensions by sprinkling iron filings or powder on a card placed directly above the magnet. Each iron particle becomes magnetized and acts like a small compass needle, with its long axis parallel to the lines of force.

The notion of lines of force can be made quantitative by defining the field strength $H$ as the number of lines of force passing through unit area perpendicular to the field. A line of force, in this quantitative sense, is called a *maxwell*.[2] Thus

$$1\,\text{Oe} = 1\ \text{line of force/cm}^2 = 1\ \text{maxwell/cm}^2.$$

[2]James Clerk Maxwell (1831–1879), Scottish physicist, who developed the classical theory of electromagnetic fields described by the set of equations known as *Maxwell's equations*.

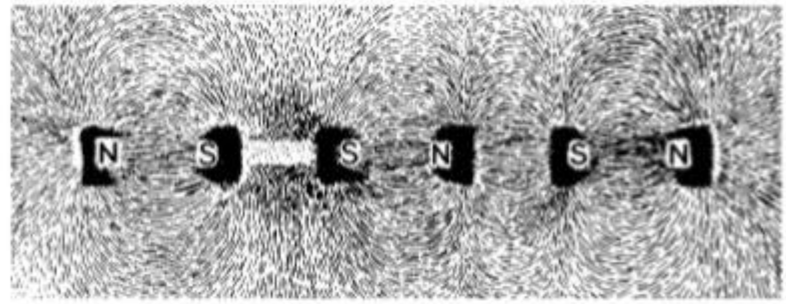


**Fig. 1.3** Fields of bar magnets revealed by iron filings.

Imagine a sphere with a radius of 1 cm centered on a unit pole. Its surface area is $4\pi$ cm$^2$. Since the field strength at this surface is 1 Oe, or 1 line of force/cm$^2$, there must be a total of $4\pi$ lines of force passing through it. In general, $4\pi p$ lines of force issue from a pole of strength $p$.

## 1.3 MAGNETIC MOMENT

Consider a magnet with poles of strength $p$ located near each end and separated by a distance $l$. Suppose the magnet is placed at an angle $\theta$ to a uniform field $H$ (Fig. 1.4). Then a torque acts on the magnet, tending to turn it parallel to the field. The moment of this torque is

$$(pH \sin \theta)\left(\frac{l}{2}\right) + (pH \sin \theta)\left(\frac{l}{2}\right) = pHl \sin \theta$$

When $H = 1$ Oe and $\theta = 90°$, the moment is given by

$$m = pl, \tag{1.4}$$

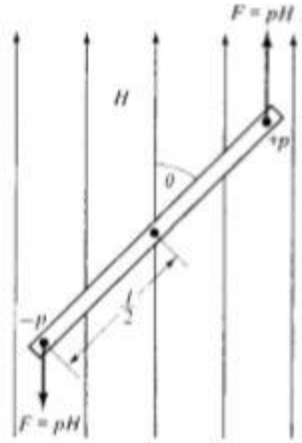


**Fig. 1.4** Bar magnet in a uniform field. (Note use of plus and minus signs to designate north and south poles.)

where $m$ is the *magnetic moment* of the magnet. It is the moment of the torque exerted on the magnet when it is at right angles to a uniform field of 1 Oe. (If the field is nonuniform, a translational force will also act on the magnet. See Section 2.13.)

Magnetic moment is an important and fundamental quantity, whether applied to a bar magnet or to the "electronic magnets" we will meet later in this chapter. Magnetic poles, on the other hand, represent a mathematical concept rather than physical reality; they cannot be separated for measurement and are not localized at a point, which means that the distance $l$ between them is indeterminate. Although $p$ and $l$ are uncertain quantities individually, their product is the magnetic moment $m$, which can be precisely measured. Despite its lack of precision, the concept of the magnetic pole is useful in visualizing many magnetic interactions, and helpful in the solution of magnetic problems.

Returning to Fig. 1.4, we note that a magnet not parallel to the field must have a certain potential energy $E_p$ relative to the parallel position. The work done (in ergs) in turning it through an angle $d\theta$ against the field is

$$dE_p = 2(pH \sin\theta)\left(\frac{l}{2}\right)d\theta = mH \sin\theta \, d\theta.$$

It is conventional to take the zero of energy as the $\theta = 90°$ position. Therefore,

$$E_p = \int_{90°}^{\theta} mH \sin\theta \, d\theta = -mH \cos\theta. \tag{1.5}$$

Thus $E_p$ is $-mH$ when the magnet is parallel to the field, zero when it is at right angles, and $+mH$ when it is antiparallel. The magnetic moment $m$ is a vector which is drawn from the south pole to the north. In vector notation, Equation 1.5 becomes

$$E_p = -\mathbf{m} \cdot \mathbf{H} \tag{1.6}$$

Equation 1.5 or 1.6 is an important relation which we will need frequently in later sections.

Because the energy $E_p$ is in ergs, the unit of magnetic moment $m$ is *erg/oersted*. This quantity is the *electromagnetic unit of magnetic moment*, generally but unofficially called simply the *emu*.

## 1.4 INTENSITY OF MAGNETIZATION

When a piece of iron is subjected to a magnetic field, it becomes magnetized, and the level of its magnetism depends on the strength of the field. We therefore need a quantity to describe the degree to which a body is magnetized.

Consider two bar magnets of the same size and shape, each having the same pole strength $p$ and interpolar distance $l$. If placed side by side, as in Fig. 1.5a, the poles add, and the magnetic moment $m = (2p)l = 2pl$, which is double the moment of each individual magnet. If the two magnets are placed end to end, as in Fig. 1.5b, the adjacent poles cancel and $m = p(2l) = 2pl$, as before. Evidently, the total magnetic moment is the sum of the magnetic moments of the individual magnets.

In these examples, we double the magnetic moment by doubling the volume. The magnetic moment per unit volume has not changed and is therefore a quantity that describes the degree to which the magnets are magnetized. It is called the *intensity of magnetization*, or

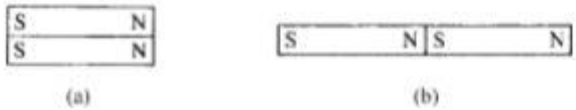


**Fig. 1.5** Compound magnets.

simply the *magnetization*, and is written $M$ (or $I$ or $J$ by some authors). Since

$$M = \frac{m}{v}, \tag{1.7}$$

where $v$ is the volume; we can also write

$$M = \frac{pl}{v} = \frac{p}{v/l} = \frac{p}{A}, \tag{1.8}$$

where $A$ is the cross-sectional area of the magnet. We therefore have an alternative definition of the magnetization $M$ as the pole strength per unit area of cross section.

Since the unit of magnetic moment $m$ is erg/oersted, the unit of magnetization $M$ is erg/oersted cm$^3$. However, it is more often written simply as emu/cm$^3$, where "emu" is understood to mean the electromagnetic unit of magnetic moment. However, *emu* is sometimes used to mean "electromagnetic cgs units" generically.

It is sometimes convenient to refer the value of magnetization to unit mass rather than unit volume. The mass of a small sample can be measured more accurately than its volume, and the mass is independent of temperature whereas the volume changes with temperature due to thermal expansion. The specific magnetization $\sigma$ is defined as

$$\sigma = \frac{m}{w} = \frac{m}{v\rho} = \frac{M}{\rho} \text{ emu/g}, \tag{1.9}$$

where $w$ is the mass and $\rho$ the density.

Magnetization can also be expressed per mole, per unit cell, per formula unit, etc. When dealing with small volumes like the unit cell, the magnetic moment is often given in units called *Bohr magnetons*, $\mu_B$, where 1 Bohr magneton $= 9.27 \times 10^{-21}$ erg/Oe. The Bohr magneton will be considered further in Chapter 3.

## 1.5 MAGNETIC DIPOLES

As shown in Appendix 1, the field of a magnet of pole strength $p$ and length $l$, at a distance $r$ from the magnet, depends only on the moment $pl$ of the magnet and not on the separate values of $p$ and $l$, provided $r$ is large relative to $l$. Thus the field is the same if we halve the length of the magnet and double its pole strength. Continuing this process, we obtain in the limit a very short magnet of finite moment called a *magnetic dipole*. Its field is sketched in Fig. 1.6. We can therefore think of any magnet, as far as its external field is concerned, as being made up of a number of dipoles; the total moment of the magnet is the sum of the moments, called dipole moments, of its constituent dipoles.

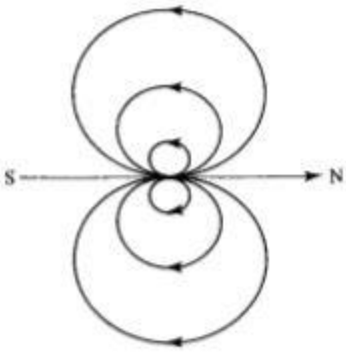


**Fig. 1.6** Field of a magnetic dipole.

## 1.6 MAGNETIC EFFECTS OF CURRENTS

A current in a straight wire produces a magnetic field which is circular around the wire axis in a plane normal to the axis. Outside the wire the magnitude of this field, at a distance $r$ cm from the wire axis, is given by

$$H = \frac{2i}{10r}\ \text{Oe}, \tag{1.10}$$

where $i$ is the current in amperes. Inside the wire,

$$H = \frac{2ir}{10r_0^2}\ \text{Oe},$$

where $r_0$ is the wire radius (this assumes the current density is uniform). The direction of the field is that in which a right-hand screw would rotate if driven in the direction of the current (Fig. 1.7a). In Equation 1.10 and other equations for the magnetic effects of currents, we are using "mixed" practical and cgs electromagnetic units. The electromagnetic unit of current, the absolute ampere or abampere, equals 10 international or "ordinary" amperes, which accounts for the factor 10 in these equations.

If the wire is curved into a circular loop of radius $R$ cm, as in Fig. 1.7b, then the field at the center along the axis is

$$H = \frac{2\pi i}{10R}\ \text{Oe}. \tag{1.11}$$

The field of such a current loop is sketched in (c). Experiment shows that a current loop, suspended in a uniform magnetic field and free to rotate, turns until the plane of the loop is normal to the field. It therefore has a magnetic moment, which is given by

$$m(\text{loop}) = \frac{\pi R^2 i}{10} = \frac{Ai}{10} = \text{amp}\cdot\text{cm}^2 \text{ or erg/Oe}, \tag{1.12}$$

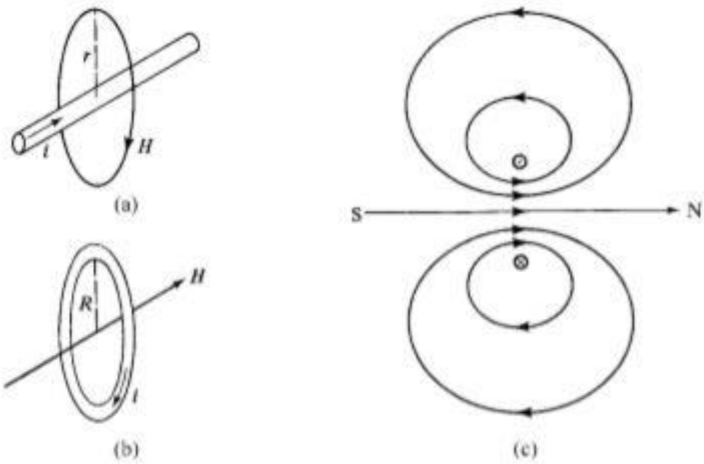


**Fig. 1.7** Magnetic fields of currents.

where $A$ is the area of the loop in $cm^2$. The direction of $m$ is the same as that of the axial field $H$ due to the loop itself (Fig. 1.7b).

A helical winding (Fig. 1.8) produces a much more uniform field than a single loop. Such a winding is called a *solenoid*, after the Greek word for a tube or pipe. The field along its axis at the midpoint is given by

$$H = \frac{4\pi ni}{10L}\ \text{Oe}, \tag{1.13}$$

where $n$ is the number of turns and $L$ the length of the winding in centimeters. Note that the field is independent of the solenoid radius as long as the radius is small compared to the length. Inside the solenoid the field is quite uniform, except near the ends, and outside it resembles that of a bar magnet (Fig. 1.2). The magnetic moment of a solenoid is given by

$$m(\text{solenoid}) = \frac{nAi}{10}\,\frac{\text{erg}}{\text{Oe}}, \tag{1.14}$$

where $A$ is the cross-sectional area.

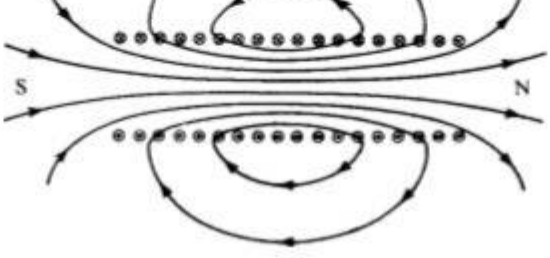


**Fig. 1.8** Magnetic field of a solenoid.

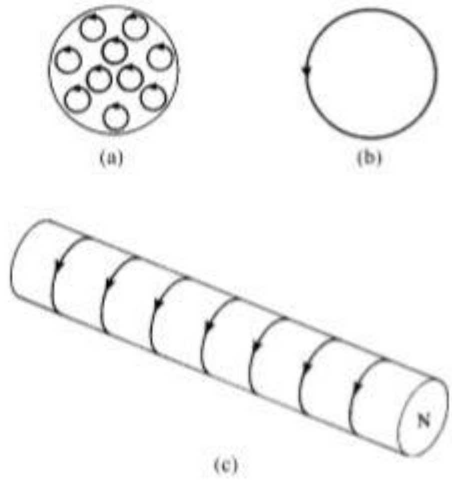


**Fig. 1.9** Amperian current loops in a magnetized bar.

As the diameter of a current loop becomes smaller and smaller, the field of the loop (Fig. 1.7c) approaches that of a magnetic dipole (Fig. 1.6). Thus it is possible to regard a magnet as being a collection of current loops rather than a collection of dipoles. In fact, André-Marie Ampère (1775–1836) suggested that the magnetism of a body was due to "molecular currents" circulating in it. These were later called *Amperian* currents. Figure 1.9a shows schematically the current loops on the cross section of a uniformly magnetized bar. At interior points the currents are in opposite directions and cancel one another, leaving the net, uncanceled loop shown in Fig. 1.9b. On a short section of the bar these current loops, called equivalent surface currents, would appear as in Fig. 1.9c. In the language of poles, this section of the bar would have a north pole at the forward end, labeled $N$. The similarity to a solenoid is evident. In fact, given the magnetic moment and cross-sectional area of the bar, we can calculate the equivalent surface current in terms of the product $ni$ from Equation 1.14. However, it must be remembered that, in the case of the solenoid, we are dealing with a real current, called a *conduction current*, whereas the equivalent surface currents, with which we replace the magnetized bar, are imaginary (except in the case of superconductors; see Chapter 16.)

## 1.7 MAGNETIC MATERIALS

We are now in a position to consider how magnetization can be measured and what the measurement reveals about the magnetic behavior of various kinds of substances. Figure 1.10 shows one method of measurement. The specimen is in the form of a ring,[3] wound with a large number of closely spaced turns of insulated wire, connected through a switch S and ammeter A to a source of variable current. This winding is called the primary, or magnetizing, winding. It forms an endless solenoid, and the field inside it is given by Equation 1.13; this field is, for all practical purposes, entirely confined to the

[3]Sometimes called a Rowland ring, after the American physicist H. A. Rowland (1848–1901), who first used this kind of specimen in his early research on magnetic materials. He is better known for the production of ruled diffraction gratings for the study of optical spectra.